Computational methods for the analysis of the dynamics of prices for storable commodities

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Abstract

Estimating a model that implements the rational expectation solution to the competitive storage model requires the computation of a numeric solution which is approximated over a finite grid of points, as introduced in a seminal paper by Deaton and Laroque (1995). This paper explores the robustness of the Pseudo Maximum Likelihood Estimator of Deaton and Laroque (1995) to alternative specifications of the grid over which to approximate the solution function. By using the same price samples that have been previously analyzed in the literature, it shows that the estimates are highly unstable when the grid is too widely spaced, thus raising doubts on their reliability. New estimates are presented which correspond to various grid specifications, to assess the sensitivity of the estimator to three crucial parameters of the grid: the lower value, the upper value and the number of grid points. Moreover, alternative specifications of the storage cost function are presented that allow for a better fit of the data sets.

Sommario

La stima del modello dello stoccaggio competitivo con aspettative razionali richiede il calcolo di una soluzione numerica approssimata
su di una griglia finita di punti, come introdotto da Deaton e Laroque (1995). Questo articolo esplora la robustezza dello stimatore PML (Pseudo Maximum Likelihood) di Deaton e Laroque a specificazioni alternative della griglia su cui approssimare la funzione che risolve il modello. Usando gli stessi dati sui prezzi che erano stati precedentemente utilizzati in letteratura, si dimostra che le stime sono altamente instabili quando la griglia utilizzata è troppo rada, avanzando quindi dei dubbi sulla loro attendibilità. Vengono presentate nuove stime che corrispondono a diverse specificazioni della griglia, per valutare la sensibilità dello stimatore a tre parametri cruciali: il limite inferiore, quello superiore ed il numero di punti. Inoltre, vengono presentate formulazioni alternative per la funzione di costo che consentono un migliore adattamento dei dati.

1 Introduction

Consider the market for a storable commodity, where supply is random, there is a fixed demand for consumption, and there exists the possibility of storing the commodity at a cost for one period, in anticipation of profits.

Theoretical models for the equilibrium of such markets have been long studied, and have come to be commonly known as “competitive storage models”. Possible solution strategies, when agents have rational expectations, had been found since the work of Gustafson (1958) (See also Samuelson 1971; Scheinkman and Schechtman 1983; Deaton and Laroque 1992).

As in other models of the same class, the solution is found by exploiting the Euler equations that characterizes the dynamic equilibrium:

\[ p_t + c(s_t) = \beta E_t[p_{t+1}], \quad \text{when } s_t > 0; \]
\[ p_t + c(s_t) \geq \beta E_t[p_{t+1}], \quad \text{when } s_t = 0 \]

(1)

where \( p_t \), the price at time \( t \), is formed under the inverse demand function depending on the level of consumption, i.e., \( p_t = p(q_t) \); \( c(s) \) is the marginal cost to store an amount \( s \) of the commodity for one period, and \( \beta \) is the one period time discount factor. In every period the accounting identity holds that:

\[ x_t \equiv h_t + s_{t-1} = q_t + s_t, \]

(2)

that is, \( x_t \), the “amount on hand”, which is formed by current production, \( h_t \), and previous period storage, \( s_{t-1} \), can be either consumed, \( q_t \), or stored for next period, \( s_t \), in anticipation of profits.

Key feature of the model is that the amount stored cannot be negative, i.e., \( s_t \geq 0, \forall t \).
One way to solve the model is to find the “equilibrium price” as a function of the amount on hand: $f(x_t)$. Consistency (i.e., rationality) of the expectations and non-negativity of storage impose that the function $f(x)$ solves the following functional:

$$f(x) = \max \left\{ E_h \left[ f(h + (x - p^{-1}(f(x)))], p(x) \right\}$$

where the expectation is taken over the possible values of the harvest, $h$, the only random component.

In general, an analytic form for the function $f(x)$ does not exist; nevertheless, it can be numerically approximated to arbitrary closeness. Numerical approximation, however, takes time and makes difficult to implement the solution to the storage model within estimation procedures. For this reason, it has been very difficult to empirically test the theory until recently.

In a series of pathbreaking papers, Deaton and Laroque opened the road to such a possibility by presenting a Pseudo Maximum Likelihood (PML) estimator for a version of the same model (Deaton and Laroque 1995, 1996).

In those articles, they found a compromise between speed and accuracy of the solution by approximating the kinked function $f(x)$ through a smooth cubic spline over a fixed grid.

This paper analyzes in some detail the consequences of such a solution strategy.

In the following, I will describe the estimator of Deaton and Laroque as originally presented and will present the results of a sensitivity analysis carried over alternative grid specifications. It will be shown that the PML estimator of Deaton and Laroque is indeed very unstable when the grid is specified as too sparse, and ways of reducing the instability will be suggested.

The discussion will be mainly technical and focused on the approximation problem, referring the interested reader to Cafiero (2002) for deeper discussion on the economics of the model and the value of alternative specifications.

2 Solution to the storage model

The simple storage model briefly introduced in the previous section can be solved by exploiting equations such as (1), which provide conditions that must hold at an equilibrium. The interpretation of equations (1) is that of a simple arbitrage condition: in presence of competitive storers, whenever stocks are held, next period price is expected to be higher than the current price by exactly the full cost of carry which includes physical storage costs and interests. If a stockout occurs (i.e., when $s_t$ hits the non negativ-
ity constraint), the arbitrage breaks down and current price can exceed the discounted, net expected price for next period.

However, the Euler conditions expressed by (1) need not to specify how expectations are formed. To actually solve the model, the expectation operator has to be given content. Gustafson (1958) described the equilibrium of a model where the action of speculators, who store commodity in anticipation of profits, is consistent—in expectations—with the outcome of the market, an example of what would later be termed as “rational expectations” by Muth (1961) and that, for our purposes, could be better described as “model consistent expectations” (see the introduction in Williams and Wright 1991).

One of the key elements of Gustafson’s model was that its solution had no analytic form. Rather, it had to be found iteratively by means of a numerical procedure—a feature common to many other rational expectations models in economics. Many algorithms have been proposed, since then, for the numerical solution of the storage model (see Wright and Williams 1984; Deaton and Laroque 1995; Deaton and Laroque 1996; Judd 1998) all of which share a common basic strategy that can be described by using the terminology of optimal control. To find a solution amounts to defining a solution function—which expresses one of the ‘controls’ of the model as a function of the ‘state’—and then approximating it with a flexible form through iterative procedures.

The natural ‘state’ for this model would be the total availability, $x_t$ that is the sum of production and carrying-in stocks from the previous period. The ‘control’ would naturally be thought of as the amount of carrying-out stocks, $s_t$, even though one could also consider, as alternative controls, the quantity consumed, $q_t$, or even the equilibrium price, $p_t$.

The solution method suggested by Gustafson (1958), and adopted later by Deaton and Laroque (1992, 1995, 1996) and by Miranda and Rui (1999), can be described as follows: Let us define the solution in terms of the price, treating it as a function of the available supply, i.e.: $p_t = f(x_t)$. If a stationary solution exists, it can be implicitly described by the following

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1Gustafson found and presented the solution for a model with no supply response, specific demand, supply and storage cost functions, but did not address the question of whether a solution to such models would always exist under more general conditions. Deaton and Laroque (1992) provide sufficient conditions for the existence of the solution to a simplified version of the model, in which storage cost is only due to physical decay of the amount stored. Scheinkman and Schechtman (1983) provide sufficient conditions for the existence of the solution to a more general model with supply response.


3See also Judd (1998, section 17.4).
functional:

\[ f(x) = \max \left\{ \beta \int f(h + (x - p^{-1}(f(x)))) g(h) dh - c(x - p^{-1}(f(x))), p(x) \right\} \]

(4)

where \( \beta = \frac{1}{1+r} \) and \( g(h) \) is the density of the random harvest process.

An equation such as (4) also suggests a natural iteration scheme to approximate the unknown function: A guess is made on the function to be approximated, which is then evaluated over a given set of grid points for \( x \). Then, the same guess is used to calculate the value of the expression to the right hand side at the same grid points, given the distribution of \( h \) and the parameters of the \( p(\cdot) \) and \( c(\cdot) \) functions. If the values of the two sides differ, the guess is updated and the iteration repeated until the function replicates itself.

The solution based on the function \( f(x) \) is not the only possibility. Williams and Wright (1991, chapter 3) describe in detail an alternative solution strategy, based on approximating the function that expresses the expected, next period price conditional on the amount of carrying-out stock, i.e., \( \psi(s_t) \equiv E_t(p_{t+1} \mid s_t) \).

Such a function is a legitimate description of the equilibrium, given that the amount of carrying-out stock, \( s_t \), completely characterizes the state of the system at the end of period \( t \) and knowledge of the \( E_t(p_{t+1}) \) allows to compute all other current optimal decisions.

While, in principle, one is free to choose any of the two suggested solution functions, in practice the choice of the function to be approximated and the method of approximation can represent a crucial element in determining the speed and the quality of the solution to the model, especially when the focus is on the econometric implementation of the model. Any such implementation will necessarily nest the solution to the storage model within some optimization procedure, and that is why accuracy and speed become crucial. If the solution is too slow, the exploration of the parameter space required by the optimization procedure might become infeasible. On the other hand, trying to gain speed by reducing the accuracy of the solution may induce intolerable instabilities in the optimization algorithm.

The solution function suggested by Gustafson needs to be approximated over the whole range of possible availabilities. Given the assumptions of the model, it will inevitably present a point of non-differentiability—a “kink”—in correspondence of the price that induces a stock-out (see equation 4), and thus strictly in the interior of the relevant range of availabilities. An

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4The algorithm had been first used by the same authors in Wright and Williams (1982), and briefly described in Wright and Williams (1984). See also Miranda and Helmberger (1988).
approximation based on a smooth function over an a priori fixed grid will have
great difficulties in trying to correctly locate the kink along the function, and
this in turn might cause problems when the model is used in an estimation
procedure especially if the location of the kink has crucial consequences on
the inference drawn from the model.

Correspondingly, one of the main advantages of the solution proposed by
Williams and Wright (1991) is precisely that the $\psi(s)$ function needs only
to be approximated over the non-negative range of $s$, thus avoiding the need
of smoothing around the kink corresponding to $s = 0$. This means that it
can be conveniently approximated by using low order polynomials or other
powerful approximation method suited for smooth functions.

Apart from the nature of the function, three other decisions are crucial
to completely define a solution strategy:

1. the choice of a flexible form for the unknown function
2. the selection of the number and location of the points of the grid over
   which to numerically approximate it, and
3. a quadrature formula for approximating the expectation on the right
   hand side of equation (1).

Each of these decisions presents various alternatives and the overall effec-
tiveness of the modeling strategy will depend crucially on the combination
of the techniques chosen at each step.

2.1 The flexible form for the function approximation

As choices of the flexible form to be used to approximate the unknown func-
tion, possible candidates include simple low order polynomials, splines, and
more sophisticated linear combinations of orthogonal polynomials.\(^5\)

If the solution to the model is not to be used for estimation, there is no
relevant advantage in choosing one or the other of the possible forms. As a
matter of fact, even a linear spline over a fixed grid of points can provide
an acceptable level of accuracy, provided a very fine grid is chosen (see for
example Gustafson 1958 or Deaton and Laroque 1992).

Things become more problematic when the solution of the model is re-
quired within an estimation procedure: in such cases, functions such as $f(x)$
or $\psi(s)$ are usually included in a broader routine that evaluates the criterion

\(^5\)For a comprehensive discussion on function approximation methods in computational
economics, see the excellent treatments in Judd (1998, chapter 6) and in Miranda and
Fackler (2002, chapter 7).
function to be maximized. Possible non-differentiabilities of the formers will carry through the latter, thus making the search for an extremum more difficult, especially if one uses optimization routines based on numerical derivatives. Furthermore, because the approximation needs to be performed every time the optimization algorithm modifies the underlying parameter values, speed of calculation becomes extremely relevant.

There is evidently a trade off between speed and accuracy. This is the main reason why it would be preferable to use smooth functions, such as polynomials or cubic splines, rather than piecewise linear or quadratic splines.

One other issue not to be overlooked in the context of the model we are discussing is that the approximated function, be it $f(x)$ or $\psi(s)$, will need to be inverted many times to map from the observed prices to the unobserved, implied harvests. If the approximated function turns out to be non monotonic over the range of the observed prices, inversion may be impossible. While an approximation the inverse of the $f(x)$ function obtained via cubic spline can be easily achieved (see below), inversion of the $\psi(s)$ function is more problematic, and slows the entire procedure considerably.\(^6\)

### 2.2 The selection of the grid size and number of points

A related issue concerns the choice of the grid over which to approximate the unknown function. Three choices affect the speed and accuracy of the solution: the size of the grid, the number of points within the grid, and how such points are distributed along the grid. For splines, in general, the finer the grid, the better is the approximation, which would suggest that, everything else constant, one should try and use a large number of grid points. However, this comes at a cost in term of the speed of the solution, which might become an issue in the estimation exercise.

When using polynomial approximations, on the other hand, it is not clear that a finer grid would yield a better approximation. There exist numerical methods for efficient approximation that use orthogonal polynomials interpolated on opportunely spaced nodes. Such methods could guarantee better approximations on limited number of nodes. Given the trade-off between speed and accuracy that necessarily must be faced in estimation exercises, these methods should be carefully considered.\(^7\)

\(^6\)As a matter of fact, Deaton and Laroque (1995, page S20) mention that failure in warranting monotonicity in the approximated function is what made them decide for a cubic spline approximation rather than a polynomial.

\(^7\)A thorough discussion on the efficiency of function approximation is beyond the scope of this article. Useful reference are, once again, Judd (1998, chapter 6) and Miranda and Fackler (2002, chapter 7).
2.3 The calculation of the expectations

One other trade off that can be exploited in order to improve the overall performance of the numerical solution to the storage problem, is in the choice of the method to calculate the expectation in equations such as (4). Such expectations needs to be calculated many many times. The choice of efficient quadrature methods, particularly suited for approximating expectations of random variables, might thus allow for a gain in speed while not sacrificing accuracy.\[8\]

3 Deaton and Laroque’s Pseudo Maximum Likelihood estimator.

In this section I will describe in some detail the procedure presented by Deaton and Laroque (1995, 1996) for estimating the structural parameter of a simple version of the model of supply and demand with competitive storage discussed above.

The authors had previously applied a Generalized Method of Moment (GMM) estimator on the same theoretical model verifying that it was indeed consistent with many of the stylized facts observable on thirteen series of storable commodities’ prices. (Deaton and Laroque 1992) The GMM estimator, however, could not identify all of the structural parameters needed to check whether the level of serial correlation observed in the actual price series could be matched or not. Mainly for this reason Deaton and Laroque embarked in the considerable task of developing a full information, structural estimator to be applied to the model, in what become the first known attempt of applying a maximum likelihood procedure to a dynamic model where the state variable is unobserved.

The model they considered is comprised of the following elements:

- a decreasing inverse linear demand function, \( p_t = p(q_t | \{a, b\}) \equiv a + bq_t \) where \( q_t \) is the level of consumption at time \( t \) and \( \{a, b\} \) are parameters to be estimated;

- production, \( h_t \), is random and distributed normally with zero mean and unit variance;\[9\]

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8See Judd (1998, chapter 7).

9The restriction is necessary to allow for estimation of the model, which would otherwise be unable to identify mean and variance of the harvest process (see Deaton and Laroque 1996, proposition 1, page 906). Allowing production to be negative is not a problem, given that the estimation is based on the price data only, which means that one is free
competitive storage incurs a cost due to proportional decay of the amount stored, with a coefficient $\delta$, so that $s_t$ amount stored at time $t$ becomes $(1 - \delta)s_t$ next period;

- storage is non negative, i.e. $s_t \geq 0$;

- the interest rate is fixed and equal to $r$

At each period in time, thus, the balancing equation (2) becomes:

$$x_t = h_t + (1 - \delta)s_{t-1} = q_t + s_t,$$

which implies that:

$$q_t = h_t + (1 - \delta)s_{t-1} - s_t. \quad (6)$$

The solution to the model is driven by the usual Euler condition on the relationship between current and expected, next period price, which becomes, in this case:

$$p_t = \gamma \mathbb{E}(p_{t+1} \mid p_t), \quad \text{when } s_t > 0;$$

$$p_t \geq \gamma \mathbb{E}(p_{t+1} \mid p_t), \quad \text{when } s_t = 0; \quad (7)$$

In (7) the full cost of carry is measured by the full discount coefficient $\gamma \equiv \frac{1 - \delta}{1 + r}$.

Provided the demand function is decreasing, and if the full cost of storage is non negative (i.e., if the full discount coefficient is less than unity), the model described above admits a rational expectations solution, which can be found conditionally on the distribution of the harvest, the fundamental random component.

Deaton and Laroque decided to solve the model by approximating the function $f(x)$ by finding the solution to the functional:

$$f(x \mid \theta) = \max \left( \gamma \sum_i f(h_i + (1 - \delta)(x - p^{-1}(f(x))))) \Pr(h_i), p(x) \right) \quad (8)$$

where $\gamma = \frac{1 - \delta}{1 + r}$ and $h_i$ and $\Pr(h_i)$ are discrete values chosen to approximate the standard normal distribution and $\theta \equiv \{a, b, \delta\}$ is the vector of unknown parameters to be estimated.

Deaton and Laroque approximated their solution function by a cubic spline over a fixed, equally spaced grid of points, a procedure that raises some to choose any scaling for the implied quantities, provided the quantity does not become infinitely negative. In the estimation exercise, such an occurrence is prevented by the finite discretization of the support of the harvest process, so that the lowest possible value for $h_t$ is always finite.
concerns because it will necessarily smooth around the point of non differentiability implied by the max operator in (8). Furthermore, the estimating procedure requires the inversion of the approximated function. Deaton and Laroque obtain such inversion “by reversing the roles of the grid and of the function, and once again using cubic splines for interpolation, now using the non-uniformly spaced grid generated by the function values” (Deaton and Laroque 1995, page S21). Such a procedure is very sensitive to some important modeling decisions, such as the number of points in the grid or the bounds of the approximated function’s domain. In order to be able to locate it, one should consider a very fine grid, at least in the range around the presumed location of the kink. A sensitivity analysis of the PML procedure of Deaton and Laroque to the number of grid points, in fact, shows extreme instability of the results as we will see in detail in the next section.

Given the maintained assumption of known independent and identical distributions of the harvests—the only random component of the model—Deaton and Laroque build the pseudo-likelihood function based on the deviations between the model consistent expected prices and the observed ones. In order to do so, they build the model consistent mean, $m(p_t)$, and variance, $v(p_t)$, of next year’s price conditional on current price, and use those values to write the function:

$$2 \log L = 2 \sum_{i}^{T-1} \log l_i = -(T - 1) \log(2\pi) - \sum_{i}^{T-1} \log v(p_i)$$

$$- \sum_{i}^{T-1} \frac{[p_{t+1} - m(p_t)]^2}{v(p_t)}.$$  

which, as they point out, is not the true (twice the) log-likelihood because prices are not distributed normally even when harvests are.

The functions $m(p)$ and $v(p)$ are built as follows:

$$m(p_t) = \sum_{i=1}^{N} f \left[ (1 - \delta)(f^{-1}(p_t) - p^{-1}(p_t)) + h_i \right] \Pr(h_i)$$

and

$$v(p_t) = \sum_{i=1}^{N} f^2 \left[ (1 - \delta)(f^{-1}(p_t) - p^{-1}(p_t)) + h_i \right] \Pr(h_i) - m^2(p).$$

The pseudo likelihood function is then maximized using an algorithm with numeric derivatives evaluated by small perturbations.\(^{10}\)

\(^{10}\)As we will see, the use of a maximization algorithm based on numeric derivatives probably forced the decision of approximating the solution to the storage model with a smooth function.
Deaton and Laroque applied their procedure to the same thirteen series of prices they had previously analyzed with the GMM model, and found estimates for twelve of them (Deaton and Laroque 1996, table I, p.911).

From the results obtained, they concluded for the inability of the simple storage model to replicate the levels of serial correlation observed in the actual time series, although the conclusion seems not to be warranted. (Cafiero 2002)

The most striking result was the reversion of the inference on the incidence of stockouts: with the GMM estimator (Deaton and Laroque 1992), stockouts were predicted as infrequent, thus suggesting that the smoothing mechanism of storage could be in place most of the times, and therefore suggesting that storage could, indeed, induce correlation in otherwise uncorrelated time series of prices (a result confirmed later by Ng 1996). On the contrary, the PML estimate implies a high occurrence of stockouts for cotton, as can be seen from the only graph presented (Deaton and Laroque 1996, figure 1, page 914, reported also in Deaton and Laroque 1995, figure 9, page S29)\textsuperscript{11}

The question of the ability of the storage model to explain the autocorrelation of time series of prices remained unresolved.

4 Re-estimation of the PML model and sensitivity analysis on the grid specification

The discrepancy between the results of the GMM estimation and of the PML estimation on the same set of data was striking. In an attempt to highlight possible shortcomings of the PML procedure as presented by Deaton and Laroque, I tried to verify whether the approximation method used in the PML model could have had consequences on the quality of the estimates.

By following the detailed description in Deaton and Laroque (1995) it has been fairly simple to replicate the estimation.\textsuperscript{12}

I have been able to virtually replicate the results for all commodities but cocoa, maize and wheat. For these three commodities, my exercise have clearly identified different maxima (see table 1).\textsuperscript{13}

\textsuperscript{11}From the graph, one can see that the level of price corresponding to a stockout, indicated as $p^*$, is estimated for cotton at a level of about 0.55. Almost half of the prices in the series are above such value. If one had to believe those estimates, there would be stockouts half of the times. In Deaton and Laroque (1992), the same parameter was estimated at 0.94, and the stockout incidence was measured to be of only eight percent. In Ng (1996), the value of $p^*$ for cotton was estimated at 0.79.

\textsuperscript{12}I have done so by writing a set of Matlab\textcopyright routines available upon request.

\textsuperscript{13}A couple of differences need to be noted that might explain the slight differences
Table 1: Estimation results of the spline- PML model

<table>
<thead>
<tr>
<th>Commodity (*)</th>
<th>Coefficients</th>
<th>$p^*$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Cocoa</td>
<td>0.141</td>
<td>-0.223</td>
<td>0.055</td>
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<tr>
<td>Cocoa (**)</td>
<td>0.161</td>
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<td>0.136</td>
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<td>Copper</td>
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<td>0.069</td>
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<td>0.169</td>
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<tr>
<td>Jute</td>
<td>0.568</td>
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<td>0.093</td>
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<td>Palm Oil</td>
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<td>0.058</td>
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<tr>
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<tr>
<td>Wheat</td>
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<td>-0.424</td>
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Deaton and Laroque’s original results

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<th>$L$</th>
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</tbody>
</table>

(*) The first panel reports the result of my estimation, carried on by using the grid specification reported in (Deaton and Laroque 1995, table I). The second panel contains the results from (Deaton and Laroque 1995, tables II and III).

(**) Values obtained with a grid of 21 instead of 20 points.
One suspicious aspect of the estimation as presented by Deaton and Laroque was the definition of the grid over which to approximate the solution function. In setting up the estimations, I followed exactly the grid specifications they reported (Deaton and Laroque 1995, table I), and have done so even though it was unclear to me why some of the grids had to be specified over ranges with lower bounds below \(-1.755\).

As pointed out by Deaton and Laroque themselves: “since inventories cannot be negative, the smallest value of \(x\) is the minimum harvest, \(Z_1\), which is therefore the minimum point on the grid” (Deaton and Laroque 1995, page S20). The minimum harvest is set at \(-1.755\) throughout the estimation, and thus it is odd that some of the grids’ lower limit had to be set at values as low as -10 (palm oil) or -30 (cocoa).

In an attempt to reconcile the large differences for cocoa, maize and wheat, I explored different grid configurations, in the belief that perhaps the values of the grid configuration in table I of Deaton and Laroque (1995) might have been incorrectly reported. And in fact, by using a different number of grid points (21 instead of 20), I obtain results for cocoa much closer to their original results\(^\text{14}\).

In the search for the correct grid specification for cocoa, maize and wheat, what really struck me was that the PML estimator turned out to be extremely sensitive to the grid specification: by changing the number of grid points by even a single unit, the estimation would change dramatically. Such sensitivity to the number and location of grid points clearly pointed to the value of exploring with alternative grid configurations.

Another result of the exploration over alternative grid specifications was to discover that some of the grid specifications generated instabilities in the pseudo likelihood function: for some of the combinations of grid limits and number of grid points, the maximization routine had to overcome discontinuity points. Notice that such instabilities would have certainly prevented any optimization routine based on gradients from locating a maximum. Recalling between my estimates and those reported by Deaton and Laroque for most of the commodities. First, Deaton and Laroque custom coded their routine for spline approximation in Gauss®, while I have used the command ‘spline’ included in the latest Matlab® distribution (The Mathworks Inc. 2001a). As for the function maximization, Deaton and Laroque used an algorithm with numerical derivatives evaluated by small perturbations, whereas I used the grid search procedure ‘fminsearch’ included in Matlab’s Optimization Toolbox (The Mathworks Inc. 2001b). Unfortunately, this prevented me from obtaining gradient information needed for the calculation of standard errors.

My choice of using a grid search routine for function maximization was guided by the recognition that the function to be maximized is likely to present many discontinuities over the range of possible parameter values.

\(^{14}\)Despite all efforts, I could not closely replicate the estimates for maize and wheat.
that Deaton and Laroque used a maximization routine based on numerical derivatives that could not handle discontinuities in the objective function, it is very likely that, to avoid such problems, they might have been forced to select unreasonably wide grids, such as those for cocoa, copper or palm oil.

As a matter of facts, 20-points grids, ranging from -30 to 15, as the one they used for cocoa, are indeed unreasonably wide spaced for this model. The fact can be appreciated if one considers that the underlying harvest process is assumed to be normal with zero mean and unit variance. A distance of 2.25 between two adjacent grid points, as implied by the grid used for this commodity, corresponds to more than two standard deviations in the scale of implied quantities.

To highlight the effects of alternative grid specifications on the results of the estimation, I have estimated the model for all possible combinations of grid limits and sizes generated by varying in unit steps:

- the number of points in the grid, from 10 to 30,
- the lower limit of the grid from, -3 to -10,
- and the upper limit of the grid from, 10 to 20

The experiment has generated a total of 1848 sets of estimates for the three parameters of the model.

The histograms in figures 1 to 3 report the distribution of the estimates for $a$, $b$ and $\delta$ for cocoa; table 2 reports some summary statistics, while table 3 lists the eight grid configurations that generated the highest values of the maximized pseudo-likelihood, all of which lead to estimates of around 0.13 for $a$, -0.26 for $b$ and 0.05 for $\delta$.

From the histograms and the statistics reported in table 2, it can be seen that the estimates vary considerably with the specification of the grid, sometime generating values far away from the prevailing ones. Moreover, for the intercept $a$ and the decay coefficient $\delta$, the distribution of the estimates is clearly bi-modal. It is interesting to compare the values obtained in the experiment with the estimates reported by Deaton and Laroque on the same data.

As can be seen, among the latter, only the measure of the slope coefficient (0.221) is included in the interquartile interval, while the estimates of the intercept (-0.162) and of the decay coefficient (0.116) are located in the tails of the empirical distributions and are very different from those of the best fitting models.

The comparison clearly demonstrates that, conditional on the particular grid structure they selected, Deaton and Laroque have identified a maximum which proves not to be robust to alternative specifications. Had they
But why is the estimation procedure so sensitive to the grid specification? The problem is likely due to the choice of approximating the kinked function that represents the solution to the storage model with a smooth function such as the cubic spline. When the grid is not very dense, such a modeling decision can generate a very bad approximation, especially around the kink point. If there are many data points in that range, it is likely that, by changing the grid only slightly, the fit of the model would radically change. The results presented by Deaton and Laroque imply, in general, high frequency of data point around what they identify as the location of the kink point as can

\[\text{Figure 1: Empirical distribution of the intercept coefficient } a \text{ for cocoa for different grid specifications}\]

\[\text{performed a more consistent search over different values for the grid boundaries and, especially, for the number of grid points, they would have discovered the presence of several maxima}^{15}.

\[\text{Deaton and Laroque report on an experiment aimed at exploring the risk of multimodality in the pseudo-likelihood function. However, it was conducted only by changing the starting values for the parameters to be estimated and by keeping the same grid specification, and thus it could not have revealed the problem.}\]
be appreciated, for example, from figure 7 of Deaton and Laroque (1995).

Table 2: Summary statistics of the estimates for cocoa for different grid specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>min</th>
<th>25%</th>
<th>median</th>
<th>75%</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.0953</td>
<td>0.1130</td>
<td>0.1322</td>
<td>0.1577</td>
<td>0.7138</td>
</tr>
<tr>
<td>$b$</td>
<td>-3.2089</td>
<td>-0.2538</td>
<td>-0.2381</td>
<td>-0.2070</td>
<td>-0.1262</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-0.0181</td>
<td>0.0452</td>
<td>0.0618</td>
<td>0.1079</td>
<td>0.2264</td>
</tr>
</tbody>
</table>

To illustrate how the approximated function changes depending upon the location of the grid points, I have plotted the approximated functions generated with the procedure suggested by Deaton and Laroque corresponding to three alternative grid specifications (figure 4), for a given value of the
Figure 3: Empirical distribution of the decay coefficient $\delta$ for cocoa for different grid specifications

Table 3: Best fitting models for cocoa

<table>
<thead>
<tr>
<th>grid specification</th>
<th>parameters</th>
<th>pseudo-L</th>
<th>$p^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
<td>$\delta$</td>
</tr>
<tr>
<td>11 -10 19</td>
<td>0.1370</td>
<td>-0.2715</td>
<td>0.0536</td>
</tr>
<tr>
<td>10 -7 19</td>
<td>0.1363</td>
<td>-0.2748</td>
<td>0.0576</td>
</tr>
<tr>
<td>10 -10 16</td>
<td>0.1378</td>
<td>-0.2677</td>
<td>0.0519</td>
</tr>
<tr>
<td>10 -4 20</td>
<td>0.1388</td>
<td>-0.2564</td>
<td>0.0496</td>
</tr>
<tr>
<td>10 -7 18</td>
<td>0.1466</td>
<td>-0.2417</td>
<td>0.0567</td>
</tr>
<tr>
<td>10 -9 14</td>
<td>0.1332</td>
<td>-0.2548</td>
<td>0.0500</td>
</tr>
<tr>
<td>11 -9 17</td>
<td>0.1335</td>
<td>-0.2626</td>
<td>0.0588</td>
</tr>
<tr>
<td>12 -9 19</td>
<td>0.1343</td>
<td>-0.2514</td>
<td>0.0496</td>
</tr>
</tbody>
</table>

unknown parameters.
As can be seen, the two functions generated by 20-point grids cannot identify the kink. On the contrary, a function generated with a 500-point grid clearly locates the point of non differentiability.

Even though the first two functions may look similar in the range of availabilities between -5 and 5, their difference can be better appreciated when plotting the inverse function generated as suggested by Deaton and Laroque over the range between 0.05 and 0.4, which covers most data points for cocoa (figure 5).

The vertical distance between the inverse price function obtained with grids of 20 points and that obtained with a grid of 500 points is excessive, when considering that these functions are used in the model for the crucial step of mapping from the observed price to the implied availability and thus to the implied harvest.

To use the function labeled as “grid limits: (-30, 15); 20 points” in figure 5 instead of the one with label “grid limits: (-5, 15); 500 points” would imply
Figure 5: Final spline approximation to the inverse of the price function for cocoa for different grid specification

an entirely different sequence of availabilities, and thus completely alter the inference. This consideration implicitly suggests that, in order to have robust estimation, the grid should be specified with a high number of points.

A grid of 500 points for cocoa have proved to be robust to grid limits ranging from -5 to -10 and from 10 to 50, thus suggesting that such a grid could be sufficiently dense.

5 Improved Pseudo Maximum Likelihood estimates

Because of the sensitivity to the grid specification, the estimates presented in the first panel of table 1 are clearly unreliable. This section collects the results of several tests aimed at improving them. First, the sensitivity to alternative grid specification is explored further. Then, estimations obtained
Table 4: Best fitting PML-spline models

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Coefficients</th>
<th>Grid size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
</tr>
<tr>
<td>Cocoa</td>
<td>0.1475</td>
<td>-0.3027</td>
</tr>
<tr>
<td>Coffee</td>
<td>0.4227</td>
<td>-1.6302</td>
</tr>
<tr>
<td>Copper</td>
<td>0.1858</td>
<td>-0.2399</td>
</tr>
<tr>
<td>Coffee</td>
<td>0.5833</td>
<td>-0.5555</td>
</tr>
<tr>
<td>Cotton</td>
<td>1.0110</td>
<td>-1.5523</td>
</tr>
<tr>
<td>Jute</td>
<td>0.5589</td>
<td>-0.4762</td>
</tr>
<tr>
<td>Maize</td>
<td>0.8293</td>
<td>-2.0429</td>
</tr>
<tr>
<td>Palm oil</td>
<td>0.5254</td>
<td>-0.3442</td>
</tr>
<tr>
<td>Rice</td>
<td>1.0538</td>
<td>-1.8516</td>
</tr>
<tr>
<td>Sugar</td>
<td>0.5799</td>
<td>-0.9621</td>
</tr>
<tr>
<td>Palm oil</td>
<td>0.9860</td>
<td>-2.5924</td>
</tr>
<tr>
<td>Rice</td>
<td>0.4474</td>
<td>-0.4850</td>
</tr>
<tr>
<td>Sugar</td>
<td>0.5270</td>
<td>-0.4826</td>
</tr>
<tr>
<td>Tea</td>
<td>0.2486</td>
<td>-1.1973</td>
</tr>
<tr>
<td>Tea</td>
<td>0.3187</td>
<td>-1.1258</td>
</tr>
<tr>
<td>Tin</td>
<td>0.4319</td>
<td>-0.2503</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.5146</td>
<td>-1.1232</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.1547</td>
<td>-0.2688</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.5384</td>
<td>-0.6898</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.6062</td>
<td>-0.7706</td>
</tr>
</tbody>
</table>

For each commodity, the first line contains the estimates that correspond to the highest value of the maximized pseudo likelihood. When this estimates imply a negative $\delta$, the second line contains the next best estimate compatible with a positive $\delta$.

with a much finer grid and varying interest rate are presented. Finally, an alternative specification of the cost of storage is explored.

5.1 Further on the sensitivity to the grid specification

By estimating the PML model with alternative combinations of grids limits and number of grid points on the same price series of data analyzed by Deaton and Laroque, it can be found that the estimates changes with the grid specification for all commodities.
The various grid specifications can be ranked in terms of the apparent fit of the data. Table 4 reports the best fitting models as judged from the values of the maximized pseudo likelihood function.

The most striking result is that, for most commodities, the estimated value for $\delta$ is negative. This apparently strange outcome is likely the consequence of having kept the interest rate fixed at 0.05 throughout the estimation. The storage model with proportional decay, in fact, can be numerically solved provided that $\frac{1-\delta}{1+r} < 1$, which implies that a condition for the solution is simply that $\delta > -r$. When keeping the interest rate fixed at five per cent, it is thus enough to constrain $\delta$ to be larger than -0.05 to ensure a solution.

Note that these results are consistent with the outcome of the application of the GMM method to the same data. Nevertheless, it is not clear that the theoretical storage model is compatible with a negative value of $\delta$, which would imply that the commodity in stock grows. For this reason, negative estimates for $\delta$ should be considered suspicious.

Interestingly, there is always some grid specification that generates estimates that are compatible with positive values of $\delta$, the best of which are also reported in table 4.\footnote{If we were to assume that these latter estimates are correct, one interesting result would be that the decay coefficients are consistently estimated at lower values than those obtained by Deaton and Laroque for all commodities. A lower value for the decay coefficient implies a lower cost of storage, and thus this new estimates would be compatible with lower occurrence of stockouts than that implied by Deaton and Laroque’s original estimates and thus potentially able to account for higher levels of serial correlation in prices.}

One other interesting result would be noted by observing the implied level of cut-off price. Column (4) of the table reports the estimated value of $p^*$, that is the maximum price consistent with the presence of stocks, so that any data point above $p^*$ would identify the occurrence of a stockout. For all of the commodities considered, the best fitting models among those consistent with at least one stockout obtain values of $p^*$ close to those obtained with the GMM method (Cafiero 2002).

All of this seems an indication that the discrepancy between the structural PML model as estimated by Deaton and Laroque and the original GMM estimates of the same model can be reduced. However, the estimates presented in table 4 are still unsatisfactory, and that is for two main reasons. First, the embarrassing abundance of maxima that can be located by moving the grid limits makes the results unreliable and raises the question of how to choose among them. The result is due to the fact that the estimates are obtained with grids of 10 to 20 points, and thus are still affected by the problem of imprecise location of the kink point.

Second, considering that some of the estimates yield negative values for
the decay coefficient, the value of 5% for the interest rate is probably too high and might be responsible for the estimated negative decay coefficients of the best fitting models. All of this leads to the attempt of estimating the model with a much finer grid and where the interest rate is allowed to vary described below.

5.2 Estimates with a finer grid and varying interest rate

The preceding discussion has led to the conclusion that the setting of the estimation model followed by Deaton and Laroque is unsatisfactory. A better estimation procedure should be based on a larger number of grid points, and possibly not to maintain a fixed interest rate.

Unfortunately, attempts at jointly estimating the interest rate and the decay coefficient have been, up to now, unsuccessful. The numerical procedure encounters unavoidable instabilities due to the impossibility of setting the appropriate grid size when both the interest rate and the decay coefficient are left to vary. While the lower limit for the grid size can be safely set to a value close to the minimum harvest, the higher limit depends on the parameters to be estimated. When the upper limit is too low, the approximated price function will not extend enough to cover the lower values in the dataset. By trial and error it is possible to identify suitable grid limits for any given value of the interest rate, but it seems impossible to find one that would prove robust to all possible values explored by the maximization routine.

Until an ad hoc grid search routine can be programmed, an alternative *modus operandi* is to perform the estimates for different fixed values of the interest rate.

Table 5 reports the results of an experiment on the data for bananas, in which estimates of the storage model are obtained by setting the grid limits to (-5, 30) and with 500 points in the grid for various values of the interest rate. Such a grid has proved to be fine enough to be able to capture the true underlying structure of the price function and not be sensitive to alternative

---

17 The theoretical limit for the upper bound to the range of availabilities is given by \( h_{\text{max}} / \delta \), where \( h_{\text{max}} \) is the maximum harvest, set here at 1.755. In fact, if harvest would remain indefinitely at its maximum, and nothing was consumed, availability would evolve according to Deaton and Laroque (1995, page S20):

\[
x_{t+1} = (1 - \delta)x_t + h_{\text{max}}.
\]

By changing the interest rate, the estimated value of \( \delta \) changes sensibly from one iteration to the next, thus compromising the appropriateness of any chosen grid size.
Table 5: Estimates of the PML model for bananas, for different values of the interest rate

<table>
<thead>
<tr>
<th>$r$</th>
<th>$a$</th>
<th>$b$</th>
<th>$\delta$</th>
<th>PL</th>
<th>$p^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.020</td>
<td>0.5806</td>
<td>-0.2008</td>
<td>0.0246</td>
<td>111.545</td>
<td>0.6251</td>
</tr>
<tr>
<td>0.025</td>
<td>0.5796</td>
<td>-0.2109</td>
<td>0.0167</td>
<td>111.978</td>
<td>0.6326</td>
</tr>
<tr>
<td>0.030</td>
<td>0.5814</td>
<td>-0.2121</td>
<td>0.0111</td>
<td>112.426</td>
<td>0.6353</td>
</tr>
<tr>
<td>0.035</td>
<td>0.5824</td>
<td>-0.2126</td>
<td>0.0055</td>
<td>112.878</td>
<td>0.6371</td>
</tr>
<tr>
<td>0.040</td>
<td>0.5838</td>
<td>-0.2138</td>
<td>-0.0001</td>
<td>113.335</td>
<td>0.6398</td>
</tr>
</tbody>
</table>

width of the grid, as confirmed by limited experiments with different grid limits.

Interest rates set at values included between 0.02 and 0.04 yielded the estimates reported in the table, which show clearly the relationship between the value of the interest rate and the decay coefficient: several combinations of $r$ and $\delta$ compete for similar levels of maximized pseudo-likelihood in explaining the data, suggesting an evident problem of identification of the two parameters.

Despite of what is implied by the theoretical model, the PML estimating model based on linear decay of stocks appears to be unable to identify the interest rate from the decay coefficient: the various rows of table 5 report virtually the same estimate of the demand function and of the full discount rate $1 - \frac{\delta}{1+r}$.

Similar results can be found for many other commodities: cocoa, coffee, cotton, jute, rice, sugar, tea and wheat, whereas no estimate compatible with a positive $\delta$ can be found for copper, maize, palm oil and tin. (Table 6)

For two of the commodities, cocoa and jute, two different maxima have been identified as witnessed by the different value of $p^*$. The value of the maximized pseudo likelihood is very similar among the different parameterizations and thus it cannot be a useful criterion for selecting the best estimates. In such cases, the tables report the two competing sets of estimates\(^{18}\).

For all other commodities, the different estimates obtained for varying interest rates identify the same maximum, as can be verified by calculating the implied value of the full storage cost coefficient, and by the closeness of

\(^{18}\)The estimates listed in the tables are those that correspond to the higher value of pseudo likelihood that I have been able to locate. Nothing ensures that a more extensive search over alternative starting values would not identify other local maxima with higher pseudo likelihood, as it could generate estimates also for the remaining commodities.
Table 6: Improved estimates of the PML model with proportional decay

<table>
<thead>
<tr>
<th>Commodity</th>
<th>$r$</th>
<th>$a$</th>
<th>$b$</th>
<th>$\delta$</th>
<th>$p^*$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bananas</td>
<td>0.03</td>
<td>0.5814</td>
<td>-0.2121</td>
<td>0.0111</td>
<td>0.6353</td>
<td>112.4</td>
</tr>
<tr>
<td>Cocoa (1)</td>
<td>0.03</td>
<td>0.1290</td>
<td>-0.2459</td>
<td>0.0894</td>
<td>0.2142</td>
<td>118.1</td>
</tr>
<tr>
<td>Cocoa (2)</td>
<td>0.05</td>
<td>0.1234</td>
<td>-0.2697</td>
<td>0.0465</td>
<td>0.2300</td>
<td>118.8</td>
</tr>
<tr>
<td>Coffee</td>
<td>0.03</td>
<td>0.1840</td>
<td>-0.3997</td>
<td>0.0493</td>
<td>0.3539</td>
<td>119.2</td>
</tr>
<tr>
<td>Cotton</td>
<td>0.045</td>
<td>0.5865</td>
<td>-0.4089</td>
<td>0.1013</td>
<td>0.6118</td>
<td>28.5</td>
</tr>
<tr>
<td>Jute (1)</td>
<td>0.03</td>
<td>0.5700</td>
<td>-0.3556</td>
<td>0.1079</td>
<td>0.5878</td>
<td>37.7</td>
</tr>
<tr>
<td>Jute (2)</td>
<td>0.05</td>
<td>0.5718</td>
<td>-0.4418</td>
<td>0.0595</td>
<td>0.6563</td>
<td>38.9</td>
</tr>
<tr>
<td>Rice</td>
<td>0.02</td>
<td>0.3562</td>
<td>-0.9809</td>
<td>0.0224</td>
<td>0.8555</td>
<td>28.6</td>
</tr>
<tr>
<td>Sugar</td>
<td>0.03</td>
<td>0.3993</td>
<td>-1.0032</td>
<td>0.1215</td>
<td>0.7395</td>
<td>-9.5</td>
</tr>
<tr>
<td>Tea</td>
<td>0.05</td>
<td>0.5107</td>
<td>-0.1683</td>
<td>0.1561</td>
<td>0.4288</td>
<td>63.9</td>
</tr>
<tr>
<td>Wheat</td>
<td>0.04</td>
<td>0.5337</td>
<td>-0.7706</td>
<td>0.0183</td>
<td>0.8560</td>
<td>29.8</td>
</tr>
</tbody>
</table>

the values of the $p^*$ parameter.

Table 6 summarizes the results by reporting one of the alternative sets of estimated parameters for each commodity. All of the estimates imply a low cost of storage, which is consistent with a lower occurrence of stockouts (as inferred from the value of $p^*$), and thus that should be consistent with higher levels of induced serial correlation of what found by Deaton and Laroque for the same commodities.

5.3 A model with constant marginal cost of storage

From the analysis presented up to now, the modeling aspect that raises more concern is the decision to model the cost of storage through proportional decay. Such formulation makes it very difficult to identify the effects of physical cost from those of the financial cost. Moreover, with proportional decay the marginal cost of storage, which determines the incentives to store, will raise as the amount of stocks falls thus increasing the probability of a stockout.

There is also evidence that a model based on the opposite assumption of a log-linear cost of storage function, according to which marginal cost of storage becomes negative as the quantity of stocks approaches zero, thus capturing a form of convenience yield, does a much better job in generating high levels of serial correlation (Miranda and Rui 1999).

For these reasons, a better modeling strategy would be to consider the
Table 7: Estimates of the PML model with constant marginal cost of storage

<table>
<thead>
<tr>
<th>Commodity</th>
<th>PL</th>
<th>$p^*$</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a$</td>
</tr>
<tr>
<td>Cocoa</td>
<td>132.6559</td>
<td>0.9452</td>
<td>0.2162</td>
</tr>
<tr>
<td>Coffee</td>
<td>132.6825</td>
<td>1.1109</td>
<td>0.2736</td>
</tr>
<tr>
<td>Copper</td>
<td>101.2113</td>
<td>1.5320</td>
<td>0.5096</td>
</tr>
<tr>
<td>Cotton</td>
<td>79.0394</td>
<td>5.3734</td>
<td>0.6749</td>
</tr>
<tr>
<td>Jute</td>
<td>56.0420</td>
<td>1.8491</td>
<td>0.7252</td>
</tr>
<tr>
<td>Maize</td>
<td>45.4387</td>
<td>1.4607</td>
<td>0.8486</td>
</tr>
<tr>
<td>Palm oil</td>
<td>71.1860</td>
<td>1.8625</td>
<td>0.6648</td>
</tr>
<tr>
<td>Rice</td>
<td>63.3137</td>
<td>6.4736</td>
<td>0.6027</td>
</tr>
<tr>
<td>Sugar</td>
<td>-2.4217</td>
<td>0.9124</td>
<td>0.6164</td>
</tr>
<tr>
<td>Tin</td>
<td>158.8119</td>
<td>0.8872</td>
<td>0.3105</td>
</tr>
<tr>
<td>Wheat</td>
<td>56.4433</td>
<td>6.3864</td>
<td>0.5313</td>
</tr>
</tbody>
</table>

marginal cost of storage as constant, as for example in Williams and Wright (1991). Such specification is intermediate between those of Deaton and Laroque (1995) and of Miranda and Rui (1999). It should capture a lower incidence of stockouts than the proportional decay formulation and, contrary to the classical supply of storage, be able to model the occurrence of stockouts. Moreover, the problems of identification between physical and financial components of the cost of storage should be reduced.

Table 7 reports the results of the estimation of such a model\textsuperscript{19}. The analysis of the figures presented in the table points to four main results. First, with constant marginal cost of storage it is possible to jointly estimate the demand function parameters, the marginal cost of storage and the interest rate, something that has proven challenging with the constant decay model.

Second, for many commodities, the estimates imply zero stockouts, as can be inferred by comparing $p^*$ with the maximum price of each commodity. The only exceptions are sugar, with 21 stockouts, maize, with 2 stockouts, and palm oil, with 1 stockout over the 88 observations.

Third, for all commodities but cocoa, coffee and sugar, the maximum of the pseudo likelihood is achieved for $r = 0.00$. This result, coupled with the low estimates of $k$, suggests low cost of storage, much lower than that implied by previous estimates of the PML model, and more in line with the classical supply of storage model of Miranda and Rui (1999).

Fourth, and final, the values of the maximized pseudo likelihood suggest

\textsuperscript{19}The storage model is solved through the spline procedure over a fine grid of 500 points, and the pseudo maximum likelihood is formed as described in the text.
that the model fits the data quite well, something that can be appreciated by comparing the values in the first column of table 7 with the corresponding ones presented by Deaton and Laroque (1995, table III, page S37). In that table, the author presented the highest values of the maximized pseudo likelihood they were able to identify with a model where autocorrelation was embedded in the harvest process (labeled as the “full model”). For seven out of the eleven commodities for which I have obtained estimates, the i.i.d storage model with constant marginal cost of storage obtains values of the maximized pseudo likelihood equal or higher to those of Deaton and Laroque’s full model, and for the remaining four they are very close. Contrary to the main conclusion of Deaton and Laroque, thus, the i.i.d. storage model is a very good candidate for serious explanation of the data, provided the correct representation of the structure of storage cost is allowed.

6 Conclusions

This paper has been concerned with a careful review of the application of the pseudo maximum likelihood estimator of the competitive storage model of Deaton and Laroque. The attempt at replicating it and a sensitivity analysis have discovered that the previous application is affected by numerical problems. The solution function at the heart of the storage model has been approximated with a flexible form in a way that has proven not to be robust to alternative specification of the range of values over which it is approximated.

By increasing the number of nodes used for the spline approximation of the kinked function that represents the model consistent price, it has been possible to get more reliable results.

For many of the commodities analyzed, the data support lower implicit cost of storage than previously found, and for some of them this allow for explaining serial correlation to a higher extent than before.

A better modeling strategy has proven to be the one that considers a fixed marginal cost for storage. By estimating the PML model with such alternative specification of the storage cost and a sufficiently dense grid, even the simple i.i.d. storage model has proved to fit the data very well and to be able to explain most of the correlation in the series of price indexes, thus confirming the results obtained by Miranda and Rui (1999) in a model with no stockouts.

The detailed exploration of the innovative estimation procedure presented by Deaton and Laroque, while confirming the huge potential that computational methods present for the analysis of dynamic models, has revealed the importance of careful consideration of the choice and use of numeric proce-
dures. The analysis has provided insights that go beyond the model studied here and apply more generally to the growing field of computational economics.

Many extensions can be imagined for the estimating model discussed here, ranging from improved numerical procedures for the solution of the storage model, such as for example the use of parameterized expectations of Williams and Wright (1982, 1984) to the use of more flexible forms for the demand function, or to the inclusion of supply response.

References


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